corresponding angular wavelength ($\pi b_o/m$) of these modes is on the order of the coupling hole size, which thus provides an extra energy loss mechanism for such surfaces modes.

V. Conclusion

A study of open coaxial resonators was addressed giving emphasis to the influence of the inner conductor geometry on the cavity selective properties. Making use of a geometrical design criterion for resonance of TE eigenmodes, a cavity was constructed and cold-tested in the frequency range 11–14 GHz. In agreement with theory, it was then demonstrated that some modes were effectively suppressed when introducing a coaxial insert of suitable shape into the empty cavity.

References

[1] M. E. Read, G. S. Nusinovich, O. Dumbrajs, H. Q. Dinh, D. Opie, G. Bird, K. Kreischer, and M. Blank, "Design of a 3 MW, 140 GHz gyrotron based on a $TE_{21,13}$ coaxial cavity," in *Proc. 18th Int. Conf. Infrared Millimeter Waves*, Colchester, England, vol. SPIE 2104, 1993, p. 521.

[2] J. J. Barroso and R. A. Correa, "Coaxial resonator for a megawatt, 280 GHz gyrotron," *Int. J. Infrared Millimeter Waves*, vol. 12, no. 7, p. 717, 1991.

[3] S. N. Vlasov, L. I. Zagrydskaya, and I. M. Orlova, "Open coaxial resonators for gyrotrons," *Radio Eng. Electron. Phys.*, vol. 21, no. 5, p. 96, 1976.

[4] P. J. Castro, J. J. Barroso, and R. A. Correa, "Cold tests of a 10 GHz gyrotron cavity," *Int. J. Infrared Millimeter Waves*, vol. 13, no. 1, p. 91, 1992.

Condition for Distortionless Transmission Line with a Nonuniform Characteristic Impedance

Jonas Lundstedt

Abstract—**The well-known condition for distortionless signal propagation on a dissipative transmission line with constant impedance is generalized to the case of nonuniform impedance. The result is based on a time-domain wave-splitting formulation of the Telegraphist's equations. It is shown that an appropriate choice of the resistance and the conductance can eliminate the distortion caused by the varying characteristic impedance. A nonuniform transmission line that satisfies the given condition is distortionless in both directions, but reflectionless for signals propagating in one direction only.**

I. Introduction

O. Heaviside derived the well-known condition for distortionless lines that states that the resistance and the conductance can be matched to each other so that the distortion vanishes on transmission lines with constant impedance. Matching of two lines with different impedance with a transmission line taper is usually done with a lossless line in order to preserve the energy of the signal at the cost of a limited bandwidth. We present a condition for distortionless *nonuniform* transmission lines. It is shown that it is possible to match the resistance and conductance to the slope of the impedance so that the signal propagates undistorted with no reflections in one direction and undistorted but with reflections in the other direction. One consequence is that it is possible to design a perfect impedance match if energy loss is acceptable. The idea to this distortionless condition has evolved from the work in [1].

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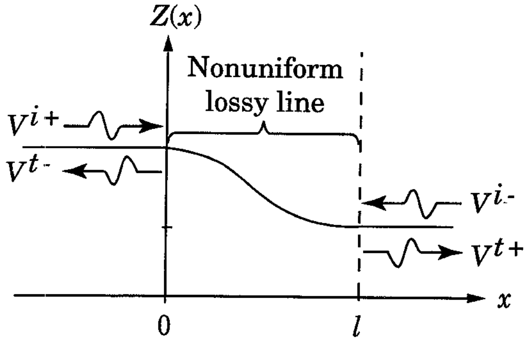

Fig. 1. The nonuniform and lossy transmission line between $x = 0$ and $x = l$ is imbedded between two uniform and lossless transmission lines.

II. The Telegraphist's Equations and the Wave-Splitting

Consider a nonuniform LCRG transmission line with length l, which is imbedded between two uniform and lossless LC transmission lines. Incident signals from the left and right side of the uniform line are denoted V^{i+} and V^{i-}, respectively. The signal generators are assumed to be impedance matched. At $x = 0$, a left-moving wave, V^{t-}, is due to transmission of the incident signal V^{i-} and reflection of V^{i+}. The corresponding right-moving wave at $x = l$ is denoted V^{t+}.

For a TEM transmission line, the voltage V and the current I satisfy the Telegraphist's equations

$$\frac{\partial}{\partial x}\begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} = \begin{bmatrix} 0 & -R(x) - L(x)\frac{\partial}{\partial t} \\ -G(x) - C(x)\frac{\partial}{\partial t} & 0 \end{bmatrix} \cdot \begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} \tag{1}$$

where $L(x), C(x), R(x)$, and $G(x)$ are respectively, the inductance, capacitance, series resistance, and shunt conductance of the line. The local characteristic impedance, $Z(x)$, and local wavefront speed, $c(x)$, are defined as

$$Z(x) = \frac{1}{Y(x)} = \sqrt{\frac{L(x)}{C(x)}}, \quad c(x) = \frac{1}{\sqrt{L(x)C(x)}}. \tag{2}$$

On a lossless and homogeneous transmission line, the solution to (1) can be decomposed into two parts, V^+ and V^-, which represent right-moving and left-moving waves, respectively

$$\begin{cases} V^+(x,t) = V^+(t - x/c) = ZI^+(t - x/c) \\ V^-(x,t) = V^-(t + x/c) = -ZI^-(t + x/c) \end{cases} \tag{3}$$

where I^+ and I^- are the currents that correspond to V^+ and V^-, respectively. The relation between V^+, V^- and the total voltage and current is

$$\begin{bmatrix} V^+(x,t) \\ V^-(x,t) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & Z(x) \\ 1 & -Z(x) \end{bmatrix} \begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} = T(x) \begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ Y(x) & -Y(x) \end{bmatrix} \begin{bmatrix} V^+(x,t) \\ V^-(x,t) \end{bmatrix} = T^{-1}(x) \begin{bmatrix} V^+(x,t) \\ V^-(x,t) \end{bmatrix}. \quad (5)$$

Since $Z(x) > 0$, the above transform between (V, I) and (V^+, V^-) is one-to-one, and it is thereby possible to use this transform also on a general nonuniform line. Then V^+ and V^- are referred to as the split voltages [2], [3]. Notice that the split voltages in (4) in general only represent physical right- and left-moving waves on those parts of a transmission line that have constant impedance and are lossless, i.e., on nonuniform lines the split voltages do not satisfy the wave equation independently.

The basic idea of wave-splitting is to find a transform from the dependent variables (V, I) to new dependent variables (V^+, V^-) based on operators that factorize the whole or part of the wave equation. Weston [4] gives a general treatment of wave-splitting and factorization for a general wave equation. However, it is usually too complicated to perform a full factorization of the wave equation, and one instead chooses a transform that factorizes the wave equation in the homogeneous media to the right and to the left of the nonuniform slab that causes the scattering. This step is justified as long as the transform is one-to-one, invertible, and factorizes the wave equation at those points where the incident, reflected, and transmitted fields are specified.

This means that the splitting can be chosen in several different ways, for example in [5], [6], one finds different splittings used for the reflected and transmitted fields from a point source above a lossy half space, and in [3], the present splitting for an LCRG line is compared to a previously considered splitting for a second order wave equation.

Thus, one normally chooses the splitting that factorizes the wave equation at the boundaries and gives simple dynamical equations for the split components. In the present case with lossless homogeneous LC lines at both boundaries, the simplest transform is that in (4) and (5). By transforming the problem from total voltages and total currents to split voltages, one obtains a formalism that is effective for large classes of transmission line direct and inverse propagation problems [1], [3], [7], [8].

Substitution of (4) and (5) into (1) gives

$$\frac{\partial}{\partial x} \begin{bmatrix} V^+(x,t) \\ V^-(x,t) \end{bmatrix} + \frac{1}{c(x)} \frac{\partial}{\partial t} \begin{bmatrix} V^+(x,t) \\ -V^-(x,t) \end{bmatrix} = \begin{bmatrix} \alpha(x) & \beta(x) \\ \gamma(x) & \delta(x) \end{bmatrix} \begin{bmatrix} V^+(x,t) \\ V^-(x,t) \end{bmatrix} \quad (6)$$

where

$$\begin{cases} \alpha(x) = \frac{1}{2}(-GZ - RY + Z_x Y) \\ \beta(x) = \frac{1}{2}(-GZ + RY - Z_x Y) \\ \gamma(x) = \frac{1}{2}(+GZ - RY - Z_x Y) \\ \delta(x) = \frac{1}{2}(+GZ + RY + Z_x Y) \end{cases} \quad (7)$$

and $Z_x Y = (\partial/\partial x) \ln Z(x)$. Notice that, for an incident signal from the left, it may seem strange that the dynamical equation (6) for $V^\pm$ on a lossy *uniform* line states that there are interactions between V^+ and V^- everywhere along the line. This peculiarity arises because V^+ is not equal to the physical right-moving wave on the lossy line, but $V = V^+ + V^-$ is. However, $V^+(l,t)$ and $V^-(0,t)$ at the boundaries are equal to the physical transmitted and reflected signals.

Since the impedance is assumed to be continuous, we get from (4) and continuity of total voltage and current that the split components $V^\pm$ are continuous in x. Thus, the incident and transmitted waves are given by the split voltages at $x = 0, l$

$$V^{i+}(t) = V^+(0,t) \quad V^{t+}(t) = V^+(l,t)$$
$$V^{i-}(t) = V^-(l,t) \quad V^{t-}(t) = V^-(0,t). \quad (8)$$

III. Distortionless Transmission Line

Assume that the incident waves $V^{i+}(t)$ arrives at $x = 0$ and $V^{i-}(t)$ arrives at $x = l$ at the time $t = 0$, i.e., for $t < 0$, $V^\pm(0 < x < l, t) = 0$, $V^+(x > l, t) = 0$, and $V^-(x < 0, t) = 0$.

By inspection of (6), one sees that the interactions between the split voltages are through the factors $\beta(x)$ and $\gamma(x)$. Therefore, let $\gamma(x)$ be zero and calculate the split voltages as functions of V^{i+} and V^{i-}. It is then convenient to introduce the wavefront travel time from x_1 to x_2

$$\tau(x_1, x_2) = \int_{x_1}^{x_2} \frac{dx'}{c(x')}. \quad (9)$$

For the wavefront travel time coordinate, one has from (9)

$$\frac{\partial}{\partial x} \tau(0,x) = \frac{1}{c(x)}, \quad \frac{\partial}{\partial x} \tau(x,l) = -\frac{1}{c(x)} \quad (10)$$

where $\tau(0,x)$ is the propagation time from the left boundary of the nonuniform line to x, and $\tau(x,l)$ is the travel time from the right boundary to x. The partial derivatives in (6) can then be reformulated as directional derivatives

$$\partial_x V^+[x, t+\tau(0,x)] = \alpha(x) V^+[x, t+\tau(0,x)] + \beta(x) V^-[x, t+\tau(0,x)] \quad (11)$$

$$\partial_x V^-[x, t+\tau(x,l)] = \delta(x) V^-[x, t+\tau(x,l)] \quad (12)$$

where $\partial_x = (\partial/\partial x)$. Equations (11) and (12) express how V^+ and V^-, respectively, varies in the direction of propagation in space-time coordinates. Since $\gamma(x) = 0$, the partial differential equation (PDE) (12) for V^- is independent of V^+, and hence, (12) is readily solved. Knowing the solution for V^-, one can then obtain the solution for V^+ from (11).

We reformulate the PDE for V^- by including the integrating factor $e^-(l,x)$

$$\partial_x \{V^-[x, t+\tau(x,l)] e^-(l,x)\} = 0 \quad (13)$$

$$e^-(x_1, x_2) = \exp\left(-\int_{x_1}^{x_2} \delta(x')\, dx'\right). \quad (14)$$

Integration of (13) from x to l, $[\tau(l,l) = 0$ and $e^-(x,l) = 1/e^-(l,x)]$

$$V^-[x, t+\tau(x,l)] = e^-(x,l) V^{i-}(t). \quad (15)$$

This means that $V^-[x, t+\tau(x,l)]$ equals the incident signal that arrives from the right multiplied by the attenuation factor $e^-(x,l)$. Next, the PDE (11) for V^+ is reformulated as an ordinary differential equation (ODE) (i.e., t is treated as a parameter) by including the integrating factor $e^+(0,x)$

$$\partial_x \{V^+[x, t+\tau(0,x)] e^+(0,x)\} = \beta(x) V^-[x, t+\tau(0,x)] e^+(0,x) \quad (16)$$

$$e^{+}(x_1, x_2) = \exp\left(-\int_{x_1}^{x_2} \alpha(x')\,dx'\right). \tag{17}$$

Substitution of (15) into (16) and integration from 0 to x gives

$$\begin{aligned} V^{+}[x, t+\tau(0,x)] = e^{+}(x,0)\Big\{ V^{\imath+}(t) + \int_0^x &\cdot \beta(x')V^{\imath-}[t + 2\tau(0,x') - \tau(0,l)] \\ &\cdot e^{-}(x',l)e^{+}(0,x')\,dx' \Big\}. \end{aligned} \tag{18}$$

It is seen that the split voltage V^+ at x is composed of the undistorted but attenuated wave $V^{\imath+}$ that impinged from the left at $x = 0$ plus a reflected and distorted part from the wave $V^{\imath-}$ that impinged from the right at $x = l$.

If we consider the case with incident waves from the left only, then according to (15), V^- is zero everywhere and V^+ is an attenuated but undistorted copy of the incident wave. Since the split voltage V^+ is the physical right-moving wave at $x > l$, the transmitted signal is truly undistorted if $V^+(l,t)$ is undistorted. It is interesting to note that in this particular case with $V^{\imath-} = 0$ and $\gamma(x) = 0, V^+$ and the corresponding $I^+ = V^+/Z$ satisfy the Telegraphist's equations and thus represent the physical right-moving wave on the nonuniform and lossy line too. V^{t+} is given by (18)

$$V^{t+}[t+\tau(0,l)] = \sqrt{\frac{Z(l)}{Z(0)}}\exp\left(-\tfrac{1}{2}\int_0^l (GZ+RY)\,dx\right)V^{\imath+}(t). \tag{19}$$

A distortionless condition for a nonuniform transmission line is hence $\gamma(x) = 0$

$$\begin{aligned} GZ - RY - Z_xY &= G\sqrt{\frac{L}{C}} - R\sqrt{\frac{C}{L}} - \frac{L_x}{2L} + \frac{C_x}{2C}. \\ &= 0 \end{aligned} \tag{20}$$

As one can see, the generalized condition differs only by the term Z_xY from Heaviside's condition, which is

$$GZ - RY = 0. \tag{21}$$

Notice that transmission through a nonuniform transmission line that satisfies (20) is distortionless for both right- and left-moving waves, but it is reflectionless only for right-moving waves, according to (15) and (18). If the line is required to be distortionless and reflectionless for waves moving in both directions, then the function $\beta(x)$ has to be zero also. However, if both β and γ are required to be zero, the only solution is that the transmission line has constant impedance. In that case, (20) reduces to the usual distortionless condition for transmission lines with constant impedance (21).

Similarly, the condition $\beta(x) = 0$ gives a distortionless nonuniform transmission line which is reflectionless for left-moving waves only.

IV. Transmitted Power on a Distortionless Transmission Line

In this section, we illustrate in some simple examples how the condition (20) affects the power distribution along the line.

Consider first the case where $V^{\imath-} = 0$ and assume for simplicity that the losses are either due to the series resistance or the shunt conductance. A tapered transmission line impedance transformer can then be made distortion- and reflectionless by adding losses. Let the impedance change from Z_0 to Z_l at the interval from 0 to l. Then the slope of the characteristic impedance causes reflections, but these reflections can be counteracted by either a resistive or a conductive loss according to (20). A resistance should be used to eliminate distortion where the impedance is decreasing, and conductance eliminates the distortion where the impedance increases. The values of R and G should be chosen according to

$$\begin{cases} Z_x < 0: & (R,G) = (-Z_x, 0) \\ Z_x > 0: & (R,G) = (0, Z_xY^2). \end{cases} \tag{22}$$

The transmitted voltage is then

$$\begin{aligned} V^{t+}&[t+\tau(0,l)] \\ &= e^{+}(l,0)V^{\imath+}(t) \\ &= \exp\left(\tfrac{1}{2}\int_0^l(-GZ - RY + Z_xY)\,dx\right)V^{\imath+}(t) \\ &= \begin{cases} Z_x < 0: & \exp\left(\int_0^l Z_xY\,dx\right)V^{\imath+}(t) \\ & \quad = \frac{Z_l}{Z_0}V^{\imath+}(t) \\ Z_x > 0: & \exp\left(\int_0^l 0\,dx\right)V^{\imath+}(t) \\ & \quad = V^{\imath+}(t). \end{cases} \end{aligned} \tag{23}$$

The transmitted power, P_t, is naturally less than the incident power, $P_\imath$, due to the losses. The ratio $P_t/P_\imath$ is

$$\begin{aligned} \frac{P_t}{P_\imath} &= \frac{(V^{t+})^2/Z_l}{(V^{\imath+})^2/Z_0} \\ &= \begin{cases} Z_l < Z_0: & \frac{(V^{\imath+}Z_l/Z_0)^2/Z_l}{(V^{\imath+})^2/Z_0} = \frac{Z_l}{Z_0} \\ Z_l > Z_0: & \frac{(V^{\imath+})^2/Z_l}{(V^{\imath+})^2/Z_0} = \frac{Z_0}{Z_l}. \end{cases} \end{aligned} \tag{24}$$

The above equations are consistent with the special case where the impedance changes abruptly from Z_0 to the smaller Z_l. The disturbances are then eliminated by connecting a series resistance $R = Z_0 - Z_l$. Notice again that the impedance transformer is only reflection- *and* distortionless for signals coming from the left.

Consider next a transmission line on which the distortion is due to losses in R and G. Equation (20) states that distortion due to R and G can be counteracted by an impedance taper. The impedance should increase where the power loss in G dominates over power loss in R and vice versa. Adjusting the impedance in this way increases the power loss in the line, but decreases the distortion. Take the stripline as an example. Let the line be situated between $x = 0$ and $x = l$ as in Fig. 1. Denote the values of Z, R and G at $x = 0$ by Z_0, R_0, and G_0, respectively, and find the function $Z(x)$ that makes the line distortionless. Assume that the thickness of the dielectric substrate is fixed and that the characteristic impedance is controlled by changing the width of stripline. Then the relations between Z, R, G and the width w in a first order approximation are $Z \propto 1/w, R \propto 1/w$, and $G \propto w$. That is, the relation between R, G, and Z are

$$R(x) = R_0Y_0Z(x) \quad \text{and} \quad G(x) = G_0Z_0/Z(x). \tag{25}$$

Equation (20) then becomes

$$\frac{Z_x(x)}{Z(x)} = G_0Z_0 - R_0Y_0 \tag{26}$$

which has the solution

$$Z(x) = Z_0\,\exp\left[(G_0Z_0 - R_0Y_0)\cdot x\right]. \tag{27}$$

Hence, a distortionless stripline, which is reflectionless for right-moving waves, should have an exponentially tapered characteristic impedance that increases if $G_0Z_0 > R_0Y_0$ and decreases if $G_0Z_0 < R_0Y_0$. The power gain follows from a derivation that is similar to that in (23) and (24), but both R and G are different from zero here. The result is

$$\frac{P_t}{P_i} = \exp\left[-(G_0Z_0 + R_0Y_0)\cdot l\right]. \tag{28}$$

V. Discussion

A new condition for distortionless nonuniform transmission lines has been developed that is a generalization of the Heaviside distortionless condition. The derivation uses the wave-splitting technique, and it is carried out in the time-domain. It is shown how the distortion can be eliminated by matching the series resistance and shunt conductance to the slope of the characteristic impedance. One should notice that the model assumes that the transmission line parameters are nondispersive. This means that if one cannot neglect dispersion, the distortionless condition can only be made valid for a limited band of frequencies. The conditions imposed on the transmission line parameters are that R, G and the slope of Z are piece-wise continuous.

References

[1] J. Lundstedt and S. Ström, "Simultaneous reconstruction of two parameters from the transient response of a nonuniform LCRG transmission line," to be published in *J. Electron. Waves and Applic.*

[2] G. Kristensson and R. J. Krueger, "Direct and inverse scattering in the time domain for a dissipative wave equation. Part I: Scattering operators," *J. Math. Phys.*, vol. 27, pp. 1667–1682, 1986.

[3] S. He and S. Ström, "Time domain wave splitting approach to transmission along a nonuniform LCRG line," *J. Electron. Waves and Applic.*, vol. 6, pp. 995–1014, 1992.

[4] V. H. Weston, "Root of a second order hyperbolic differential operator and wave splitting," in *Invariant Imbedding and Inverse Problems,* J. P. Corones, G. Kristensson, P. Nelson, and D. L. Seth, Eds. SIAM, 1992, pp. 139–152.

[5] S. He, "Factorization of a dissipative wave equation and the Green functions technique for axially symmetric fields in a stratified slab," *J. Math. Phys.*, vol. 33, pp. 953–966, 1992.

[6] S. He and A. Karlsson, "Time domain Green function technique for a point source over a dissipative stratified half-space," *Radio Sci.*, vol. 28, pp. 513–526, 1993.

[7] S. He, "Green functions technique for nonuniform LCRG transmission lines with frequency-dependent parameters," *J. Electron. Waves and Applic.*, vol. 7, pp. 31–48, 1993.

[8] J. Lundstedt and S. He, "Signal restoration after transmission through a nonuniform LCRG line," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 11, pp. 2087–2092, 1994..

New Model of Coupled Transmission Lines

Adam Abramowicz

Abstract—**The paper shows that an existing description of coupled transmission lines is inconsistent and proposes a new model based truly on the mutual coupling concept. In the existing formulation a series electric coupling and parallel magnetic coupling are combined. In the new formulation the parallel electric and magnetic couplings as well as series electric and magnetic couplings are used. Obtained model of coupled lines has physical background related to the odd and even type of propagation and agrees with the practical results.**

I. Introduction

When two unshielded uniform TEM transmission lines of the same impedance Z are located in close proximity, they become electromagnetically coupled via their associated electric and magnetic fields. Two coupled lines can be excited in the two ways: "even mode" excitation or "odd mode" excitation, i.e., in-phase or opposite-phase, equal-amplitude excitations. The characteristic impedances Z_{0e} and Z_{0o} associated with these modes are defined as the input impedance of an infinite length of one line, in the presence of (and thus electromagnetically coupled to) the second line, also of infinite length, when both are excited in the appropriate manner. A knowledge of Z_{0e} and Z_{0o} as functions of line parameters is essential to the design of filters, directional couplers, and related devices, because the coupling coefficient between lines can be calculated from them. As it has been shown in [1] the coupling coefficient k between two coupled lines when they are properly terminated can be calculated from the following formula:

$$k = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}. \tag{1}$$

Lines are properly terminated when the matching impedance Z_0 is taken as

$$Z_0 = \sqrt{Z_{0e}Z_{0o}}. \tag{2}$$

The impedance Z_0 is always less than the impedance of single line Z (without coupling), thus four mentioned impedances satisfy the following inequality:

$$Z_{0o} < Z_0 < Z < Z_{0e}. \tag{3}$$

All four impedances can be simply expressed in terms of the capacitance per unit length of the particular transmission line in question: if this parameter is denoted by C (F/m), then

$$Z\sqrt{\varepsilon_r} = \frac{1}{vC} \tag{4}$$

where: v is the velocity of light in free space and ε_r is the dielectric constant of the medium filling the line.

It should be also noted that for the uniform coupled lines the velocity of light is the same for odd or even excitations and equal to the velocity of light in the single (uncoupled) line.

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